



# Fermi National Accelerator Laboratory

FTUV/91-52 IFIC/91-51  
FERMILAB-Conf-91/335-A  
UM TH 91-31  
November 1991

## CONSTRAINTS ON FERMION MIXING WITH EXOTICS\*

Enrico Nardi<sup>♡</sup>, Esteban Roulet<sup>♣</sup> and Daniele Tommasini<sup>♣</sup>

<sup>♡</sup>*Randall Laboratory of Physics, University of Michigan  
Ann Arbor, MI 48109-1120, U.S.A.*

<sup>♣</sup>*NASA/Fermilab Astrophysics Center, Fermi National Accelerator Laboratory  
P.O. Box 500, Batavia, IL 60510-0500, U.S.A.*

<sup>♣</sup>*Departament de Física Teòrica, Universitat de València  
46100 Burjassot, Valencia, Spain*

### ABSTRACT

We analyze the constraints on the mixing angles of the standard fermions with new heavy particles with exotic  $SU(2) \times U(1)$  quantum number assignments (left-handed singlets or right-handed doublets), that appear in many extensions of the electroweak theory. The updated Charged Current and Neutral Current experimental data, including also the recent  $Z$ -peak measurements, are considered. The results of the global analysis of all these data are then presented.

November 1991

---

\* *Talk delivered at the International Workshop on Electroweak Physics Beyond the Standard Model, 2-5 October 1991, Valencia*



The existence of new, exotic fermions (i.e., which have non canonical  $SU(2)_L$  assignments), is predicted in most of the extensions of the SM. The lack of observation of new particles in the last accelerator runs indicates that these non-standard fermions should have large masses ( $> 50 - 100$  GeV), assumed they exist. Moreover, cosmological and astrophysical arguments<sup>1</sup> imply that these heavy particles cannot be stable if they carry a colour or electric charge, and should then decay into known particles. A natural decay channel could be provided by a mixing between the ordinary and exotic fermions, which is allowed whenever their  $SU(3)_C \times U(1)_{em}$  quantum numbers are the same. This mixing would also induce small deviations from the standard values of the light fermion couplings to the weak gauge bosons, that could be indirectly detected even if the exotic particles cannot be directly produced with the experimental facilities available at present.

Here we will discuss the constraints that can be set on the ordinary-exotic mixing angles from a global analysis of the present accelerator data<sup>2</sup>. The first (pre-LEP) global analysis of this kind of new physics was done by Langacker and London<sup>3</sup>. Subsequently, two of us<sup>4</sup> showed that the very first LEP data already improved some bounds significantly, more recently, Langacker, Luo and Mann<sup>5</sup> have also discussed the sensitivity to some exotic mixings that will be attained with the foreseeable precision of the ongoing or planned precision electroweak experiments.

We consider a fermion to be ordinary (exotic) if it is a left-handed  $SU(2)_L$  doublet (singlet) or a right-handed  $SU(2)_L$  singlet (doublet). Exotic fermions can appear in mirror models in which generally whole mirror generations with  $R$ -doublets and  $L$ -singlets are introduced, in models with vector doublets (singlets) where both left and right fermions have the same transformation properties under weak-isospin, or as singlet Weyl neutrinos.

The general formalism to describe fermion mixing was also introduced in ref.3. To prevent dangerous FCNC's (that are strongly constrained experimentally) each ordinary charged fermion is allowed to mix just with a single exotic state. This simplifying assumption allows us to neglect a large number of intergenerational mixing parameters. Each light mass eigenstate  $f$  then acquires an exotic component, which is  $SU(2)_L$  singlet for the left handed component  $f_L$ , and  $SU(2)_L$  doublet for the right handed component  $f_R$ . Clearly this modifies the couplings of the light fermions with the weak gauge bosons: the weak-isospin coupling  $t_3^f$  of  $f_L$  is reduced by the factor  $(c_L^f)^2 \equiv (\cos(\theta(f_L)))^2$ , and a weak-isospin coupling  $t_3^f (s_R^f)^2 \equiv t_3^f (\sin(\theta(f_R)))^2$  for  $f_R$  is induced, where  $\theta(f_{L,R})$  are the ordinary-exotic mixing angles.

For the neutral fermions the situation is more complicated because in the presence of Majorana mass terms three kinds of neutral fields with different weak isospin assignments ( $-1/2, 0, +1/2$ ) can mix at the same time, and also because due to the lack of experimental constraints the assumption on the absence of FCNC must be released. In the processes that we consider, however, a sum has to be taken over the unobserved final neutrino mass eigenstates (the kinematical effects of  $\nu$  masses are negligible), and this allows to describe the observed rates in terms of one effective

mixing angle  $\theta(\nu_i)$  per each neutrino flavour. For instance, the decay rate of the  $Z$  boson into undetected neutrinos is proportional to the sum of the squares of the neutrino neutral-current couplings<sup>2</sup>,

$$\Gamma_{inv} \propto 3 - \sum_i \Lambda_i (s_L^{\nu_i})^2 + O(s^4), \quad (1)$$

where  $\Lambda_i$  is an effective parameter lying between 0 and 4, depending on the kind of heavy neutrinos involved in the mixing. If the light states are mixed with additional ordinary states (that will be mainly heavy) then the neutral-current couplings are not affected and  $\Lambda_i = 0$ . If only singlet states mix with the known neutrinos then  $\Lambda_i = 2$  while  $\Lambda_i = 4$  describes mixings involving only  $t_3 = -1/2$  exotic states. We see that the effective parameter  $\Lambda_i$  could largely influence the reduction in the decay rate (equivalently, in the effective number of neutrino species). We note that the LEP measurement of the number of light neutrino species implies that if additional neutrinos with large doublet components exist, their masses must be heavier than  $M_Z/2$ . Light singlets, however, as in the case of Dirac neutrino masses, could be present and a mixing with exotic doublets would allow them to couple to the  $Z$  boson, thus opening a new invisible decay channel. For simplicity we do not consider this case, and assume the light neutrinos to be mainly ordinary states.

Let us now turn to the measurements that we have used to constrain the fermion mixing angles. Our set of fundamental input parameters consists of the QED coupling constant  $\alpha$  measured at  $q^2 = 0$ , the mass of the  $Z$  boson  $M_Z = 91.175 \pm 0.021$  GeV<sup>6</sup>, and the Fermi coupling constant  $G_F$ . In contrast with  $\alpha$  and  $M_Z$ , the Fermi coupling constant as extracted from the measured life-time of the  $\mu$ -lepton,  $G_\mu = 1.16637(2) \times 10^{-5} \text{ GeV}^{-2}$ , is affected by fermion mixings. The relation between  $G_F$  and the effective  $\mu$ -decay coupling constant is  $G_\mu = G_F c_L^{\nu_e} c_L^{\nu_\mu} c_L^e c_L^\mu$ . Clearly, this indirect dependence on the light lepton mixing angles propagates in all the expressions that contain  $G_F$ . This is the case for example for the  $W$  boson mass, for which no other explicit dependence on mixings appears. In addition, also the values of the top-quark  $m_t$  and Higgs boson  $M_H$  masses must be specified, since they enter the expressions via loop corrections. The dependence on  $M_H$  is soft, and we keep its value fixed at 100 GeV. In contrast, varying the value of  $m_t$  can induce sizeable effects. We have chosen to fix the top mass at the value  $m_t = 120$  GeV that corresponds approximately to the minimum of our  $\chi^2$  function when all the mixing parameters are set to zero.

For our analysis we have used the updated measurements of the  $W$ -boson mass, the Charged Current (CC) constraints on lepton universality, on Cabibbo-Kobayashi-Maskawa (CKM) unitarity, on Right Handed Currents (RHC's) as well as the Neutral Current (NC) constraints from neutrino scattering, atomic parity violation, and the recent results from  $Z$ -peak measurements. The theoretical expressions for the relevant observables are modified by the mixings, e.g. the ratios of the  $SU(2)_L$ -couplings  $g_e, g_\mu, g_\tau$  of the different lepton flavours, which test CC

universality, become  $(g_i/g_e)^2 \simeq (c_L^i)^2(c_L^{\nu_i})^2/(c_L^e)^2(c_L^{\nu_e})^2$  for  $i = \mu, \tau$ . These ratios are measured in  $W$  decay at colliders, in  $\mu$  and  $\tau$  decays, and in leptonic meson decays. The averages are  $(g_\mu/g_e)^2 = 1.014 \pm 0.010$ ,  $(g_\tau/g_e)^2 = 0.955 \pm 0.030$ , both  $\sim 1.5\sigma$  off the SM value. This disagreement could be due to problems in the experimental normalizations in the analysis of  $\tau$  decays, but it could also be a hint for new physics.

TABLE I. Results on  $Z$ -partial widths (in MeV) and on-resonance asymmetries.

Quantity	Experimental value	Correlation			
$\Gamma_Z$	$2487 \pm 10$	0.52	0.52	0.29	0.25
$\Gamma_h$	$1739 \pm 13$	-0.15	0.55	0.48	
$\Gamma_e$	$83.2 \pm 0.6$		-0.08	-0.07	
$\Gamma_\mu$	$83.4 \pm 0.9$			0.26	
$\Gamma_\tau$	$82.8 \pm 1.1$				
$A_e^{\text{FB}}(\text{peak})$	$-0.019 \pm 0.014$				
$A_\mu^{\text{FB}}(\text{peak})$	$0.0070 \pm 0.0079$				
$A_\tau^{\text{FB}}(\text{peak})$	$0.099 \pm 0.096$				
$A_\tau^{\text{pol}}$	$-0.121 \pm 0.040$				
$\Gamma_b$	$367 \pm 19$				
$\Gamma_c$	$299 \pm 45$				
$A_b^{\text{FB}}$	$0.123 \pm 0.024$				
$A_c^{\text{FB}}$	$0.064 \pm 0.049$				

Let us discuss in more detail the LEP data, which are the most important new ingredient of our analysis. The partial decay width of the  $Z$ -boson to  $f$ -flavour fermions is proportional to  $a_f^2 + v_f^2$ , while the on resonance forward-backward asymmetry  $A_f^{\text{FB}}$  is sensitive to the product  $v_f a_f$ , where

$$\begin{aligned}
v_f &= t_3^f \left[ (c_L^f)^2 + (s_R^f)^2 \right] - 2Q^f s_{eff}^2(f) \\
a_f &= t_3^f \left[ (c_L^f)^2 - (s_R^f)^2 \right]
\end{aligned} \tag{2}$$

are the vector and axial-vector  $Zf\bar{f}$  couplings (analytical formulæ for the effective weak sines  $s_{eff}(f)$  can be found in ref.<sup>7</sup>). The combined measurement of these two sets of the partial widths and of the asymmetries allows for an independent determination of  $v_f$  and  $a_f$ , and turns out to be quite effective for constraining both the right and left mixing angles even in the “joint fits” where all the mixings are allowed to be present simultaneously. Table I gives the LEP results for the partial widths and on resonance FB asymmetries into leptons (using the flavour-dependent values since we do not assume universality), and into charm and bottom. In the same we present also the value of the  $\tau$  polarization asymmetry<sup>8</sup>. In our analysis we have also included the leptonic asymmetries measured within  $\pm 1$  GeV around resonance, and the  $b$  and  $c$  asymmetries measured in the  $\gamma$ - $Z$  interference region at PEP and PETRA.

To obtain the constraints on the mixing parameters  $s_i^2$  we have confronted the theoretical expression  $X_\alpha^{th}$  for each observable with the corresponding experimental result  $X_\alpha^{exp} \pm \sigma_\alpha$  by constructing a  $\chi^2$  function

$$\chi^2 = \sum_{\alpha, \beta} \frac{(X_\alpha^{th} - X_\alpha^{exp})}{\sigma_\alpha} (C^{-1})_{\alpha\beta} \frac{(X_\beta^{th} - X_\beta^{exp})}{\sigma_\beta} \quad (3)$$

where  $C$  represents the matrix of correlations. For each parameter we then assume a probability distribution

$$P(s_i^2) = N_i e^{-\chi^2(s_i^2)/2} \quad (4)$$

with  $N_i^{-1} = \int_0^1 \exp(-\chi^2(s_i^2)/2) ds_i^2$ . For the joint fits, in which all mixing parameters are allowed to vary simultaneously, the  $\chi^2$  function in the expression for  $P(s_i^2)$  is minimized with respect to all the remaining parameters for each value of  $s_i^2$ .

The 90% c.l. upper bounds  $\bar{s}_i^2$  are computed by requiring

$$\int_0^{\bar{s}_i^2} P(s_i^2) ds_i^2 = 0.90 \quad (5)$$

under the additional condition  $\chi^2(\bar{s}_i^2) > \chi^2(0)$  that, if not satisfied, would be a signature for non-zero mixing angles at 90% c.l..

Although there are more than 20 mixing parameters, the large number of observables allows us to constrain all of them. The inclusion of the recent results from LEP, together with the updated NC and CC results, have considerably improved almost all the previous limits [3,4]. Our results<sup>2</sup> for the 90 % c.l. bounds obtained in the individual and joint analyses are collected in table II.

For simplicity we have assumed  $\Lambda_e = \Lambda_\mu = \Lambda_\tau$  (corresponding to ordinary-exotic mixings of the same kind for the three neutrinos) but these parameters could in principle differ. In the individual analysis only the bounds on the neutrino mixings can depend on the value of  $\Lambda$  and we just show the representative results for  $\Lambda = 2$ . Furthermore, since the electron and muon neutrino mixings are mainly

TABLE II. 90 % c.l. upper bounds on the ordinary-exotic fermion mixings for the individual fits, where only one parameter is allowed to vary, and for the joint fits where cancellations between different mixings can occur. The theoretical ranges corresponding to see-saw models for the mixings with exotics of mass  $\sim 100$  GeV are listed in the last column.

	Individual	Joint			Theoretical range
		$\Lambda = 2$	$\Lambda = 0$	$\Lambda = 4$	
$(s_L^e)^2$	0.0047	0.015	0.0090	0.015	$2.5 \times 10^{-11} < s^2 < 5 \times 10^{-6}$
$(s_R^e)^2$	0.0062	0.010	0.0082	0.010	
$(s_L^\mu)^2$	0.0017	0.0094	0.0090	0.011	$10^{-6} < s^2 < 10^{-3}$
$(s_R^\mu)^2$	0.0086	0.014	0.014	0.013	
$(s_L^\tau)^2$	0.011	0.017	0.015	0.017	$3 \times 10^{-4} < s^2 < 2 \times 10^{-2}$
$(s_R^\tau)^2$	0.011	0.012	0.014	0.012	
$(s_L^u)^2$	0.0045	0.019	0.015	0.019	$10^{-8} < s^2 < 10^{-4}$
$(s_R^u)^2$	0.018	0.024	0.025	0.024	
$(s_L^d)^2$	0.0046	0.019	0.016	0.019	$10^{-8} < s^2 < 10^{-4}$
$(s_R^d)^2$	0.020	0.030	0.028	0.029	
$(s_L^s)^2$	0.011	0.038	0.039	0.041	$2 \times 10^{-6} < s^2 < 1.5 \times 10^{-3}$
$(s_R^s)^2$	0.36	0.67	0.63	0.74	
$(s_L^c)^2$	0.013	0.040	0.042	0.042	$2 \times 10^{-4} < s^2 < 1.5 \times 10^{-2}$
$(s_R^c)^2$	0.029	0.097	0.10	0.099	
$(s_L^b)^2$	0.011	0.070	0.072	0.069	$2.5 \times 10^{-3} < s^2 < 5 \times 10^{-2}$
$(s_R^b)^2$	0.33	0.39	0.40	0.39	
$(s_L^{\nu_e})^2$	0.0097	0.015	0.016	0.014	
$(s_L^{\nu_\mu})^2$	0.0019	0.015	0.0087	0.011	
$(s_L^{\nu_\tau})^2 \dagger$	0.032	0.064	0.097	0.035	

$\dagger$  For a discussion of the bounds on  $s_L^{\nu_\tau}$ , see text.

constrained by CC measurements, they are largely independent of the value of  $\Lambda$ . In contrast, for the  $\tau$  neutrino different values of  $\Lambda$  led to different bounds, since in this case the LEP measurement of  $\Gamma_Z$  gives an important constraint. The *upper* bounds for  $\nu_\tau$  are respectively  $(s_L^{\nu_\tau})^2 < 0.098, 0.032, 0.015$  for  $\Lambda_\tau = 0, 2, 4$  corresponding to neutrino mixings with heavy ordinary doublets in sequential or vector doublets, with heavy singlets and with exotic doublets respectively. For the joint bounds, we present all the results for  $\Lambda = 0, 2$  and  $4$ .

If only one mixing angle is considered at a time, for most of the mixing factors  $(sf)^2$  the limits are below the 1 % level, with the exception of the mixings of  $u_R$ ,  $d_R$ ,  $c_R$  and  $\nu_{\tau L}$  that are of the order of a few percent and those of  $s_R$  and  $b_R$  that are still poorly constrained to values  $\lesssim 1/3$ . In the joint fits, where accidental cancellations among different mixings can occur, the constraints are relaxed by a factor between 2 and 5.

Some peculiarities arise in the  $\tau$ - $\nu_\tau$  sector, due to the observed deviation of the  $\tau$ -lepton decay rate from the SM prediction, on which we have commented. Non-vanishing  $\tau_L$  and/or  $\nu_{\tau L}$  mixings weaken the  $W\tau\nu_\tau$  coupling, allowing for a longer  $\tau$  lifetime, as is favoured by experiments. The excellent agreement of the accurate LEP measurements with the SM predictions forces the overall probability distribution to be consistent with vanishing values for the  $\tau$  and  $\nu_\tau$  mixings. However, if  $\nu_\tau$  mainly mixes with an ordinary sequential or vector doublet neutrino ( $\Lambda_\tau \simeq 0$ ), the NC experiments are ineffective for constraining this mixing. In this case, both in the individual and in the joint analyses, we find that the value  $s_L^{\nu_\tau} = 0$  falls out of the 90% confidence regions, that are respectively  $0.0075 < (s_L^{\nu_\tau})^2 < 0.098$  and  $0.0057 < (s_L^{\nu_\tau})^2 < 0.097$ . However, within two standard deviations the data are consistent with zero mixing.

In the last column of table II we list, for each mixing angle  $\theta(f)$ , the theoretical range obtained assuming that the mixing is generated by a see-saw mechanism either quadratic or linear in the ordinary-to-exotic masses ratio:  $(m_f/M_f)^2 < \sin^2 \theta(f) < m_f/M_f$ , where  $m_f$  is the mass of the ordinary fermion considered, and  $M_f$  is the mass of the exotic fermion involved in the mixing. For the heavy masses we have taken a common mass scale  $M_f = 100$  GeV. We see that for the  $\tau$  lepton, and for the  $c$  and  $b$  quark, we are now starting to test the theoretically interesting region.

Two of us (E.N. and D.T.) thank José Valle and Fernando Campos Carvalho for their very nice hospitality at the Valencia Meeting. The work of E. R. was supported by the DOE and NASA (grant NAGW# 1340) at Fermilab.

## References

1. A. de Rújula, S. Glashow and U. Sarid; Nucl. Phys. B333 (1990) 173;  
E. Nardi and E. Roulet, Phys. Lett. B245 (1990) 105.
2. E. Nardi, E. Roulet and D. Tommasini, SISSA 104/91/EP (FERMILAB 91/207-A).
3. P. Langacker and D. London, Phys. Rev. D38 (1988) 886.
4. E. Nardi and E. Roulet, Phys. Lett. B248 (1990) 139.
5. P. Langacker, M. Luo and A.K. Mann, UPR-458T (1991).
6. ALEPH Collaboration, D. Decamp et al., CERN-PPE/91-105;  
DELPHI Collaboration, P. Abreu et al., CERN-PPE/91-95;

- L3 Collaboration, B. Adeva et al., L3 Prep. # 028 (1991);  
OPAL Collaboration, M. Alexander et al., CERN-PPE/91-81.
7. M. Consoli and W. Hollik, 'Z physics at LEP 1' vol. 1, eds. G. Altarelli et al., CERN 89-08;  
G. Burgers and F. Jegerlehner, *ibidem*;  
W. Hollik, CERN-TH.5661/90 (FEB. 1990);  
G. Burgers and W. Hollik, in 'Polarization at LEP' vol. 1, eds. G. Alexander et al., CERN 88-06;  
B. W. Lynn, M. E. Peskin and R. G. Stuart, in 'Physics at LEP' vol. 1, eds. J. Ellis and R. Peccei CERN 86-02 (1986);  
D.C. Kennedy and B.W. Lynn, Nucl. Phys. B 322 (1989) 1;  
J.G. Im, D.C. Kennedy, B.W. Lynn and R.G. Stuart, Nucl. Phys. B 321 (1989) 83;  
J.L. Rosner, EFI 90-18 (June 1990).
8. ALEPH Collaboration, D. Decamp et al., CERN-PPE/91-94;  
OPAL Collaboration, G. Alexander et al., CERN-PPE/91-103.